SOLVING ABSOLUTE VALUE AND QUADRATIC INEQUALITIES

10.3.3

There are several methods for solving absolute value and quadratic inequalities, but one method that works for all kinds of inequalities is to change the inequality to an equation, solve it, then put the solution(s) on a number line. The solution(s), called "boundary point(s)," divide the number line into regions. Check any point within each region in the *original* inequality. If that point is true, then all the points in that region are solutions. If that point is false, then none of the points in that region are solutions. The boundary points are included in ($\geq$ *or* $\leq$) or excluded from ($>$ *or* $<$) the solution depending on the inequality sign.

Example 1

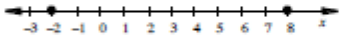
Solve: $|x-3| \leq 5$

Change to an equation and solve.

$$|x-3| = 5$$

$$x-3=5 \text{ or } x-3=-5$$

$$x=8 \text{ or } x=-2 \text{ (the boundary points)}$$



Choose $x=-3$, $x=0$, and $x=9$ to test in the original inequality. $x=-3$ is false, $x=0$ is true, and $x=9$ is false.

false true false

The solution is all numbers greater than or equal to -2 and less than or equal to 8, written as $-2 \leq x \leq 8$.

Example 2

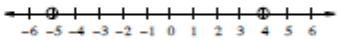
Solve: $|2y+1| > 9$

Change to an equation and solve.

$$|2y+1| = 9$$

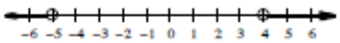
$$2y+1=9 \text{ or } 2y+1=-9$$

$$y=4 \text{ or } y=-5 \text{ (the boundary points)}$$



Choose $y=-6$, $y=0$, and $y=5$ to test in the original inequality. $y=-6$ is true, $y=0$ is false, and $y=5$ is true.

true false true



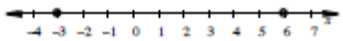
The solution is all numbers less than -5 or numbers greater than 4, written as $y<-5$ or $y>4$.

Example 3

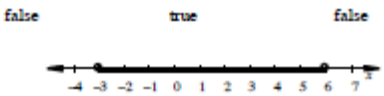
Solve: $x^2 - 3x - 18 < 0$

Change to an equation and solve.

$$x^2 - 3x - 18 = 0$$
$$(x-6)(x+3) = 0$$
$x = 6$ or $x = -3$ (the boundary points)



Choose $x = -4$, $x = 0$, and $x = 7$ to test in the original inequality. $x = -4$ is false, $x = 0$ is true, and $x = 7$ is false.



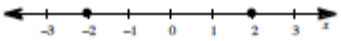
The solution is all numbers greater than 3 and less than 6, written as $-3 < x < 6$.

Example 4

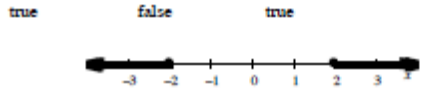
Solve: $m^2 - 3 \geq 1$

Change to an equation and solve.

$$m^2 - 3 = 1$$
$$m^2 = 4$$
$m = \pm 2$ (the boundary points)



Choose $m = -3$, $m = 0$, and $m = 3$ to test in the original inequality. $m = -3$ is true, $m = 0$ is false, and $m = 3$ is true.



The solution is all numbers less than or equal to -2 or all numbers greater than or equal to 2, written as $m \leq -2$ or $m \geq 2$.

Problems

Solve each inequality.

1. $|x+4| \geq 7$
2. $x^2 + 6x + 8 < 0$
3. $y^2 - 5y > 0$
4. $|x| - 5 \leq 8$
5. $|x-5| \leq 8$
6. $y^2 - 5y < 0$
7. $|4r-2| > 8$
8. $x^2 - 3x - 4 < 0$
9. $|3x| \leq 12$
10. $-x^2 - 9x - 14 < 0$
11. $|1-3x| \leq 13$
12. $y^2 \leq 16$
13. $|2x-3| > 15$
14. $3x^2 + 7x - 6 \geq 0$
15. $|5x| > -15$
16. $-2|x-3| + 6 < -4$
17. $x^2 + 4x - 8 < 4$
18. $y^2 + 6y + 9 > 0$
19. $|4-d| \leq 7$
20. $x(7x-26) \leq 8$

Answers

1. $x \geq 3$ or $x \leq -11$
2. $-4 < x < -2$
3. $y < 0$ or $y > 5$
4. $-13 \leq x \leq 13$
5. $-3 \leq x \leq 13$
6. $0 < y < 5$
7. $r < -\frac{3}{2}$ or $r > \frac{5}{2}$
8. $-1 < x < 4$
9. $-4 \leq x \leq 4$
10. $x < -7$ or $x > -2$
11. $-4 < x < \frac{14}{3}$
12. $-4 \leq y \leq 4$
13. $x < -6$ or $x > 9$
14. $x \leq -3$ or $x \geq \frac{2}{3}$
15. all numbers
16. $x > 8$ or $x < -2$
17. $-6 < x < 2$
18. all numbers except -3
19. $-3 \leq d \leq 11$
20. $-\frac{2}{7} \leq x \leq 4$